

What is claimed is:

- 1 1. A method for reconstructing an image of a scattering medium, comprising:
2 directing energy into the scattering medium at a source location on the
3 scattering medium;
4 measuring the energy emerging from the scattering medium at a detector
5 location on the scattering medium;
6 selecting an initial guess of internal properties of the scattering medium;
7 predicting the energy emerging from the scattering medium using an
8 equation of radiative transfer, wherein the prediction is a function of the initial guess;
9 generating an objective function based on a comparison of the prediction
10 with the measurement;
11 generating a gradient of the objective function by a method of adjoint
12 differentiation;
13 modifying the initial guess of the properties of the scattering medium
14 based on the gradient of the objective function; and
15 generating an image representation of the internal properties of the
16 scattering medium.

- 1 2. The method according to claim 1, further comprising repeating the
2 predicting of the energy emerging from the scattering medium based on the modified
3 initial guess, generating the objective function and modifying the initial guess, until at
4 least one of a predetermined number of repetitions has occurred and the objective
5 function reaches a predetermined threshold.

1 3. The method according to claim 1, wherein the prediction depends on the
2 boundary conditions.

1 4. The method according to claim 3, wherein the boundary conditions
2 account for a refractive mismatch at an interface between the medium and at least one of
3 the detectors and source.

1 5. The method according to claim 1, wherein the prediction comprises an
2 iterative process producing intermediate results.

1 6. The method according to claim 5, wherein the intermediate results of the
2 prediction are stored.

1 7. The method according to claim 6, wherein generating the gradient of the
2 objective function by adjoint differences uses the intermediate results of the prediction.

1 8. The method according to claim 7, wherein generating the gradient
2 comprises stepping backward through the intermediate results of the prediction.

1 9. The method according to claim 1, wherein the equation of radiative
2 transfer is time independent.

1 10. The method according to claim 9, wherein the time independent equation
2 of radiative transfer is:

$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

where $\Psi(\mathbf{r}, \omega)$ is the radiance at the spatial position \mathbf{r} directed into a unit solid angle ω , $S(\mathbf{r}, \omega)$ is the energy directed into the medium at spatial position \mathbf{r} into a unit solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and $p(\omega, \omega')$ is the scattering phase function.

11. The method according to claim 10, wherein the scattering phase function is:

$$p(\cos \theta) = \frac{1 - g^2}{2(1 + g^2 - 2g \cos \theta)^{3/2}}$$

where θ is the angle between the two unit solid angles ω and ω' , and g is the anisotropy factor.

12. The method according to claim 1, wherein the equation of radiative transfer is time dependent.

13. The method according to claim 12, wherein the time dependent equation of radiative transfer is:

$$\frac{1}{c} \frac{\partial \Psi(\mathbf{r}, \omega, t)}{\partial t} = S(\mathbf{r}, \omega, t) - \omega \cdot \nabla \Psi(\mathbf{r}, \omega, t) - (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega, t) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega', t) d\omega'$$

where $\Psi(\mathbf{r}, \omega, t)$ is the radiance at the spatial position \mathbf{r} directed into a unit solid angle ω , $S(\mathbf{r}, \omega, t)$ is the energy directed into the medium at spatial position \mathbf{r} into a

6 unit solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and
7 $p(\omega, \omega')$ is the scattering phase function.

1 14. The method according to claim 13, wherein the scattering phase function
2 is:

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$$p(\cos\theta) = \frac{1 - g^2}{2(1 + g^2 - 2g\cos\theta)^{3/2}}$$

4 where θ is the angle between the two unit solid angles ω and ω' , and g is
5 the anisotropy factor.

1 15. The method according to claim 1, wherein the properties include at least
2 one of a scattering coefficient, an absorption coefficient, an anisotropy factor, and a
3 scattering phase function.

1 16. The method according to claim 1, wherein the objective function is a
2 normalized comparison of the predicted energy and the measured energy

1 17. The method according to claim 1, wherein the objective function is based
2 on the normalized sum of the differences between the predicted energy and the measured
3 energy for each source detector pair, wherein a source detector pair is formed between
4 each source location and each detector location.

1 18. The method according to claim 1, wherein the objective function is:

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$$\phi = \frac{1}{2} \sum_i^m (P_i - M_i)^2$$

3 where M_i represents the actual measurements and the P_i represents the
4 predicted measurements for each source defector pair i , m is the number of source
5 detector pairs, where a source detector pairs is formed between each source location and
6 each detector location.

1 19. The method according to claim 1, further comprising minimizing the
2 objective function.

1 20. The method according to claim 19, wherein minimizing the objective
2 function includes a one dimensional line search.

1 21. The method according to claim 20, wherein the one dimensional line
2 search is performed along a direction of the gradient of the objective function.

1 22. The method according to claim 20, wherein the one dimensional line
2 search is performed along a gradient-dependent direction.

1 23. The method according to claim 1, wherein the energy comprises near
2 infra-red energy.

1 24. The method according to claim 1, wherein the scattering medium contains
2 regions wherein the scattering coefficients are not substantially greater than the
3 absorption coefficients.

1 25. The method according to claim 1, wherein the scattering medium contains
2 a low scattering region embedded in a high scattering region.

1 26. The method according to claim 1, wherein the predicted energy is
2 determined using finite element methods.

1 27. The method according to claim 1, wherein the predicted energy is
2 determined using finite difference methods.

1 28. A method for imaging the spatial optical properties of tissue, comprising:
2 (a) directing energy into the scattering medium at a source location on
3 the tissue;
4 (b) measuring the energy emerging from the scattering medium at a
5 detector location on the tissue;
6 (c) selecting and initial guess of the spatial optical properties of the
7 tissue;
8 (d) predicting the energy emerging from the tissue using an equation
9 of radiative transfer in an iterative process, wherein the prediction is a function of the

initial guess and a refraction index mismatch at a boundary of the tissue, and the iterative process generates a plurality of intermediate predictions;

(e) generating an objective function based on a normalized comparison of the prediction with the measured energy emerging from the scattering medium;

(f) generating a gradient of the objective function by adjoint differentiation;

(g) modifying the initial guess of the spatial properties of the tissue based on the gradient of the objective function;

(h) repeating steps (d) through (g) until at least one of a threshold of modifications to the initial guess is reached and the objective function reaches a threshold; and

(j) generating an image representation of the spatial optical properties of the tissue.

29. A system for reconstructing an image of a scattering medium, comprising:
a source for directing energy into the scattering medium at source location on the scattering medium;
a detector for measuring the energy emerging from the scattering medium at a detector location on the scattering medium;
an initial guess of internal properties of the scattering medium;
means for predicting the energy emerging from the scattering medium using an equation of radiative transfer, wherein the prediction is a function of the initial guess;

9 means for generating an objective function based on a comparison of the
10 prediction with the measurement;
11 means for generating a gradient of the objective function by a method of adjoint
12 differentiation;
13 means for modifying the initial guess of the properties of the scattering medium
14 based on the gradient of the objective function; and
15 means for generating an image representation of the internal properties of the
16 scattering medium.

1 30. The system according to claim 1, further comprising means for repeating
2 the predicting of the energy emerging from the scattering medium based on the modified
3 initial guess, generating the objective function and modifying the initial guess, until at
4 least one of a predetermined number of repetitions has occurred and the objective
5 function reaches a predetermined threshold.

1 31. The system according to claim 1, wherein the prediction depends on the
2 boundary conditions.

1 32. The system according to claim 31, wherein the boundary conditions
2 account for a refractive mismatch at an interface between the medium and at least one of
3 the detectors and source.

1 33. The system according to claim 1, wherein the prediction comprises an
2 iterative process producing intermediate results.

1 34. The system according to claim 33, wherein the intermediate results of the
2 prediction are stored.

1 35. The system according to claim 34, wherein generating the gradient of the
2 objective function by adjoint differences uses the intermediate results of the prediction.

1 36. The system according to claim 35, wherein generating the gradient
2 comprises stepping backward through the intermediate results of the prediction.

1 37. The system according to claim 1, wherein the equation of radiative
2 transfer is time independent.

1 38. The system according to claim 37, wherein the time independent equation
2 of radiative transfer is:

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$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

4 where $\Psi(\mathbf{r}, \omega)$ is the radiance at the spatial position \mathbf{r} directed into a unit solid
5 angle ω , $S(\mathbf{r}, \omega)$ is the energy directed into the medium at spatial position \mathbf{r} into a unit
6 solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and $p(\omega, \omega')$
7 is the scattering phase function.

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1 47. The system according to claim 1, further comprising minimizing the
2 objective function.

1 48. The system according to claim 47, wherein minimizing the objective
2 function includes a one dimensional line search.

1 49. The system according to claim 48, wherein the one dimensional line
2 search is performed along a direction of the gradient of the objective function.

1 50. The system according to claim 49, wherein the one dimensional line
2 search is performed along a gradient-dependent direction.

1 51. The system according to claim 50, wherein the energy comprises near
2 infra-red energy.

1 52. The system according to claim 1, wherein the scattering medium contains
2 regions wherein the scattering coefficients are not substantially greater than the
3 absorption coefficients.

1 53. The system according to claim 1, wherein the scattering medium contains
2 a low scattering region embedded in a high scattering region.

1 54. The system according to claim 1, wherein the predicted energy is
2 determined using finite element methods.

1 55. The system according to claim 1, wherein the predicted energy is
2 determined using finite difference methods.

1 56. A system for imaging the spatial distribution of optical properties of
2 tissue, comprising:

3 (a) a source for directing energy into the scattering medium at a source
4 location on the tissue;

5 (b) a detector for measuring the energy emerging from the scattering medium
6 at a detector location on the tissue;

7 (c) an initial guess of spatial optical properties of the tissue;

8 (d) means for predicting the energy emerging from the tissue using an
9 equation of radiative transfer in an iterative process, wherein the prediction is a function
10 of the initial guess and a refraction index mismatch at a boundary of the tissue, and the
11 iterative process generates a plurality of intermediate predictions;

12 (e) means for generating an objective function based on a normalized
13 comparison of the prediction with the measured energy emerging from the scattering
14 medium;

15 (f) means for generating a gradient of the objective function by adjoint
16 differentiation;

17 (g) means for modifying the initial guess of the spatial properties of the tissue
18 based on the gradient of the objective function;

19 (h) means for repeating steps (d) through (g) until at least one of a threshold
20 of modifications to the initial guess is reached and the objective function reaches a
21 threshold; and

22 (j) means for generating an image representation of the spatial optical
23 properties of the tissue.

1 57. Computer executable software code stored on a computer readable
2 medium, the code for reconstructing an image of a scattering medium, comprising:
3 code to direct energy into the scattering medium at a source location on
4 the scattering medium;
5 code to measure the energy emerging from the scattering medium at a
6 detector location on the scattering medium;
7 code to receive an initial guess of internal properties of the scattering
8 medium;
9 code to predict the energy emerging from the scattering medium using an
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;
11 code to generate an objective function based on a comparison of the
12 prediction with the measurement;
13 code to generate a gradient of the objective function by a method of
14 adjoint differentiation;

15 code to modify the initial guess of the properties of the scattering medium
16 based on the gradient of the objective function; and
17 code to generate an image representation of the internal properties of the
18 scattering medium.

1 58. Computer executable software code stored on a computer readable
2 medium, the code for imaging the spatial distribution of optical properties of tissue,
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of
19 modifications to the initial guess is reached and the objective function reaches a
20 threshold; and
21 (j) code to generate an image representation of the spatial optical properties of the
22 tissue.

1 59. A computer readable medium having computer executable software code
2 stored thereon, the code for reconstructing an image of a scattering medium, comprising:
3 code to direct energy into the scattering medium at a source location on
4 the scattering medium;
5 code to measure the energy emerging from the scattering medium at a
6 detector location on the scattering medium;
7 code to receive an initial guess of internal properties of the scattering
8 medium;
9 code to predict the energy emerging from the scattering medium using an
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;
11 code to generate an objective function based on a comparison of the
12 prediction with the measurement;
13 code to generate a gradient of the objective function by a method of
14 adjoint differentiation;
15 code to modify the initial guess of the properties of the scattering medium
16 based on the gradient of the objective function; and

17 code to generate an image representation of the internal properties of the
18 scattering medium.

1 60. A computer readable medium having computer executable software code
2 stored thereon, the code for imaging the spatial distribution of optical properties of tissue,
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of
19 modifications to the initial guess is reached and the objective function reaches a
20 threshold; and

- 21 (j) code to generate an image representation of the spatial optical properties of the
- 22 tissue.

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